Dynamically Emergent Correlations

Satya N. Majumdar

Laboratoire de Physique Théorique et Modèles Statistiques, CNRS,
Université Paris-Saclay, France

Happy b'day Peter!

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Non-intersecting Brownian walkers and Yang–Mills theory on the sphere

Peter J. Forrester [a], Satya N. Majumdar [b], Grégory Schehr [c,*]

[a] Department of Mathematics and Statistics, The University of Melbourne, Victoria 3010, Australia
[b] Univ. Paris Sud, CNRS, LPTMS, UMR 8626, Orsay F-91405, France
[c] Univ. Paris Sud, CNRS, LPT, UMR 8627, Orsay F-91405, France

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Reunion Probability of N Vicious Walkers: Typical and Large Fluctuations for Large N

Grégory Schehr · Satya N. Majumdar · Alain Comtet ·
Peter J. Forrester

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Plan

- **Dynamically emergent correlations** between independent particles via **stochastic** fluctuation of the environment

  $\Longrightarrow$ **general setting**

- A simple model: **Switching** confining potential (box/trap)

- Exact **nonequilibrium stationary state (NESS)**

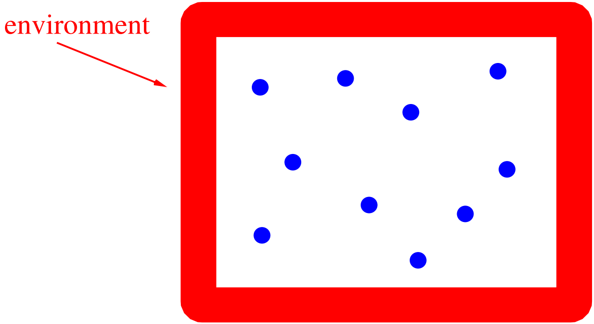
$\Longrightarrow$ **Conditionally independent and identically distributed (CIID)**

- Recent experiments using optical tweezers

- Generalisation to other models with **stochastically driven** environment

- Summary and Conclusion

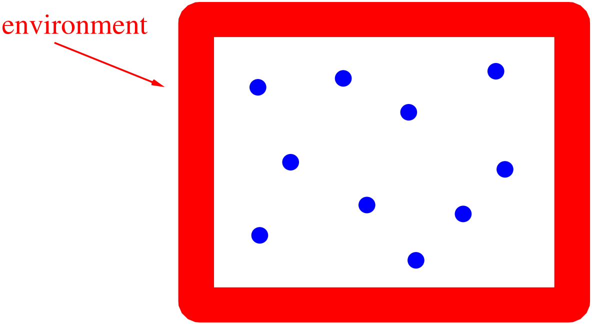
Dynamically emergent correlations: general setting



Particles: non-interacting (say Brownian)

Environment: stochastic (independent of particle motions)

Dynamically emergent correlations: general setting



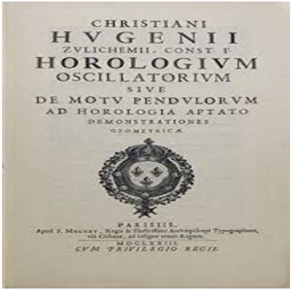
Particles: **non-interacting** (say Brownian)

Environment: **stochastic** (independent of particle motions)

Stochastic dynamics of the **environment** induces **strong correlations** between particles

$\Longrightarrow$ **Emergent correlations**

Christiaan Huygens (1629-1695)



CHRISTIANI
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DEMONSTRATIONES
GEOMETRICÆ.

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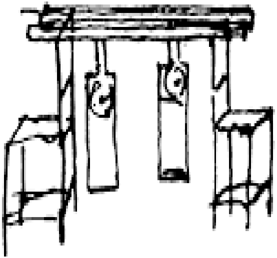
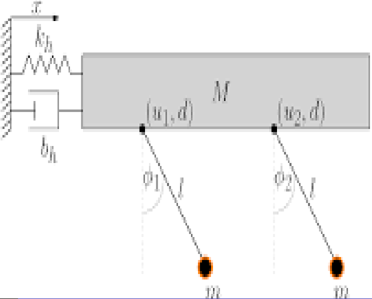
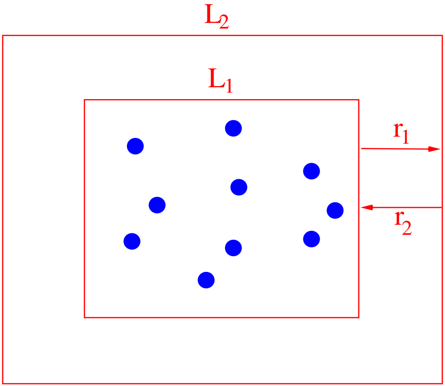
CVM PRIVILEGIO REGIS.





A simple model: stochastically switching box size



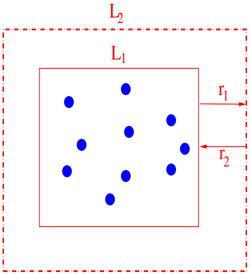
Particles ⇒ **independent** Brownian with reflecting bc. at the walls

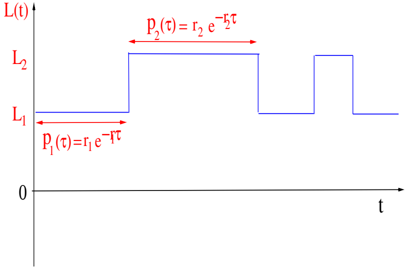
Box size ⇒ **stochastically** switching between two values L_1 and L_2

$$\mathbf{L_1} \xrightarrow{r_1} \mathbf{L_2}$$

$$\mathbf{L_1} \xleftarrow{r_2} \mathbf{L_2}$$

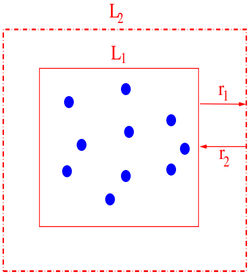
A simple model: stochastically switching box size

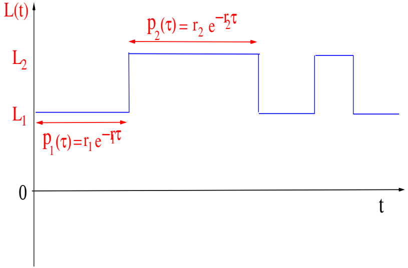




Box size $L(t)$ $\Longrightarrow$ a **dichotomous telegraphic** process

A simple model: stochastically switching box size





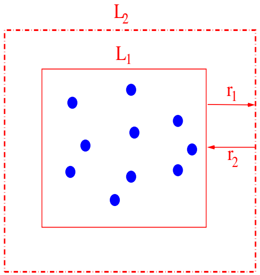
Box size $L(t) \Longrightarrow$ a **dichotomous telegraphic** process

Goal: to compute the joint PDF $P(\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_N}|t)$ at time t

In particular, the stationary state (if it exists)

$$\boxed{P^{\rm st}(\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_N}) = \mathbf{P}(\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_N}|\mathbf{t} \to \infty)}$$

An interesting limiting case

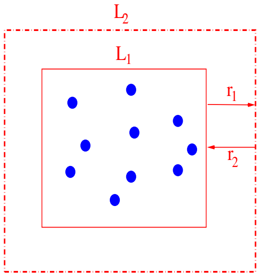


A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

An interesting limiting case

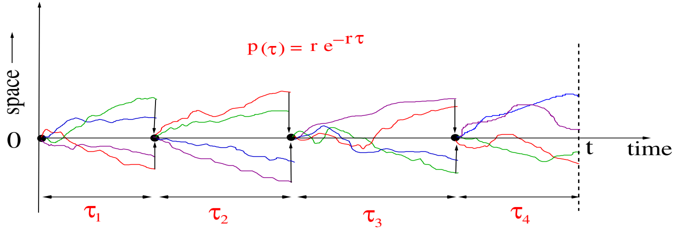


A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

N independent Brownian motions $\Longrightarrow$ **simultaneously reset** (**instantaneously**) to the origin with rate **r**



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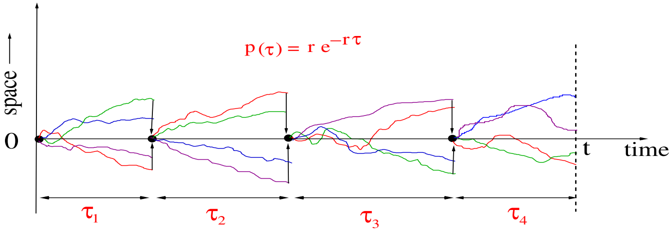

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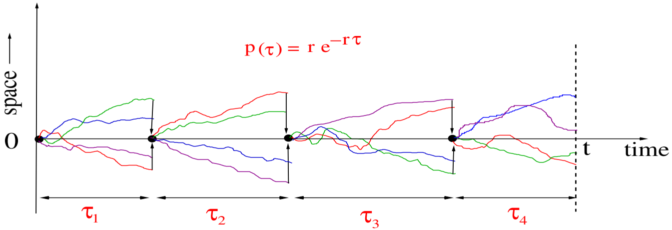
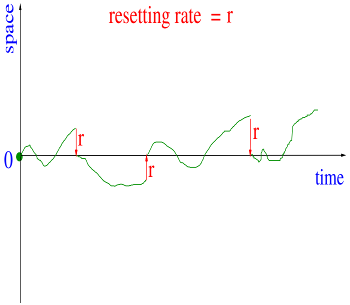


$\Longrightarrow$ A **multiparticle** generalization of **stochastic resetting** of a **single** $\mathbf{N = 1}$ diffusing particle

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Single particle $(N = 1)$

$N = 1 \Rightarrow$ a single Brownian particle in $d = 1$



Poissonian resetting

Time intervals between successive resettings distributed as:

$$p(\tau) = r\, e^{-r\tau}$$

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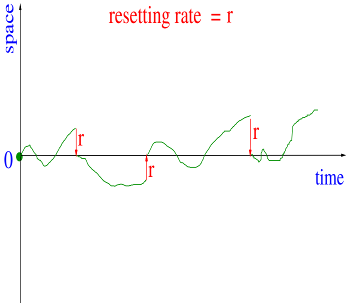

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Dynamics: In a small time interval Δt

$$x(t+\Delta t) = 0 \quad \text{with prob. } r\Delta t \quad \text{(resetting)}$$

$$= x(t) + \eta(t)\,\Delta t \quad \text{with prob. } 1 - r\Delta t \quad \text{(diffusion)}$$

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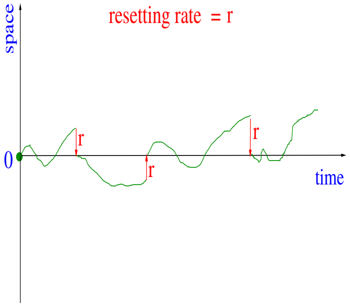

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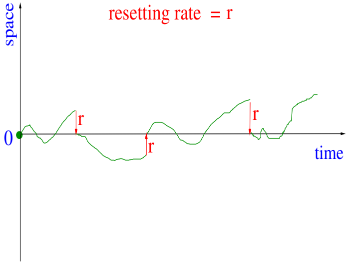
$\eta(t) \to$ Gaussian white noise: $\langle \eta(t) \rangle = 0$ and $\langle \eta(t)\eta(t') \rangle = 2\,D\,\delta(t-t')$

[M.R. Evans & S.M., PRL, 106, 160601 (2011)]

Prob. density $p_r(x,t)$ with resetting rate $r > 0$



$p_r(x,t) \to$ prob. density at time t,
given $p_r(x,0) = \delta(x)$

- In the absence of resetting ($r = 0$):

$$p_0(x,t) = \frac{1}{\sqrt{4\pi\, D\, t}}\, \exp[-x^2/4Dt]$$

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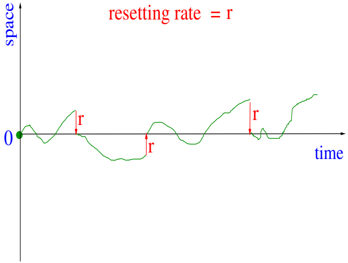

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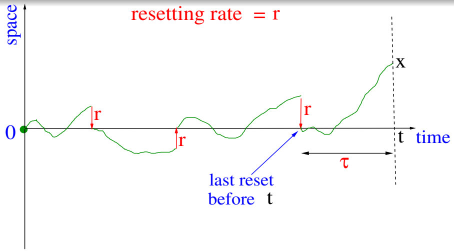
- In the absence of resetting ($r=0$):

$$p_0(x,t) = \frac{1}{\sqrt{4\pi\, D\, t}}\ \exp[-x^2/4Dt]$$

- In the presence of resetting ($r>0$):

$$p_r(x,t) = ?$$

Exact solution valid at all times t



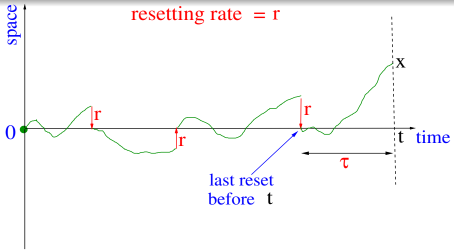
- Exact solution at all times t:

$$p_r(x|t) = e^{-r\,t}\, p_0(x|t) + \int_0^t d\tau\, (r\, e^{-r\,\tau})\, p_0(x|\tau)$$

where $p_0(x|\tau) =$ diffusion propagator $= \frac{1}{\sqrt{4\pi\, D\, \tau}}\, \exp[-x^2/4D\tau]$

Renewal interpretation: $\tau \rightarrow$ time since the last resetting during which $\Longrightarrow$ free diffusion

Exact solution valid at all times t



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Renewal interpretation: $\tau \rightarrow$ time since the last resetting during which $\Longrightarrow$ free diffusion

- As $t \to \infty$, $p_r^{\rm st}(x) = r\,\int_0^\infty p_0(x|\tau)\, e^{-r\,\tau}\, d\tau = \frac{\alpha_0}{2}\, \exp[-\alpha_0\,|x|]$ where $\alpha_0 = \sqrt{r/D}$

Stationary State

Exact solution $\rightarrow$ $\boxed{p_r^{\rm st}(x) = \frac{\alpha_0}{2}\, \exp[-\alpha_0\, |x|]}$ with $\alpha_0 = \sqrt{r/D}$

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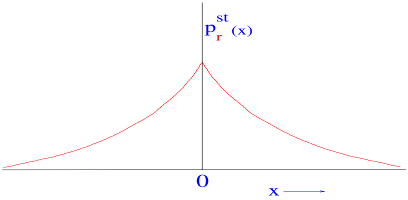

→ nonequilibrium stationary state (NESS)

⇒ current carrying with detailed balance → violated

$p_r^{\rm st}(x) = \alpha_0\, \exp[-V_{\rm eff}(x)]$

effective potential: $\alpha_0|x|$

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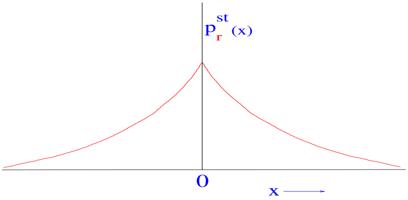

$\rightarrow$ nonequilibrium stationary state (NESS)

$\Rightarrow$ current carrying with detailed balance $\rightarrow$ violated

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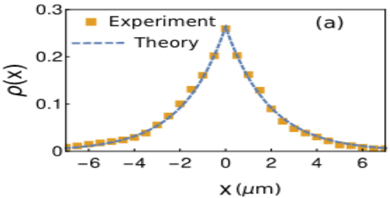

Experimental verificaion using holographic optical tweezers

Tal-Friedman, Pal, Sekhon, Reuveni, & Roichman
J. Phys. Chem. Lett. 11, 7350 (2020)

Toy model $\Longrightarrow$ explosion of activities

- Enzymatic reactions in biology (Michaelis-Menten reaction)
- Diffusion in a confining potential/box
- Lévy flights, Lévy walks, fractional BM with resetting
- Space-time dependent resetting rate $r(x, t)$
- Search via nonequilibrium reset dynamics vs. equilibrium dynamics
- Resetting dynamics of extended systems
- Memory dependent reset
- Quantum dynamics with reset
- Active particles with reset
- Cost of resetting
- Optimization of random search algorithms
- Optimal strategy for animal movements

... $\Longrightarrow$ a long list **!**

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Reviews: "Stochastic resetting and applications",

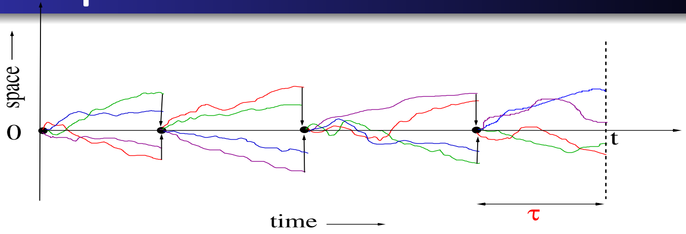
M.R. Evans, S.M., & G. Schehr, J. Phys. A. : Math. Theor. 53, 193001 (2020)

"The inspection paradox in stochastic resetting",

A. Pal, S. Kostinski & S. Reuveni, J. Phys. A. : Math. Theor. 55, 021001 (2022)

Multiparticle Generalization (N $>$ 1)

$N > 1$ independent Brownian motions in $d = 1$



Consider N Brownian motions (independent) that are **simultaneously** reset with rate r to the origin

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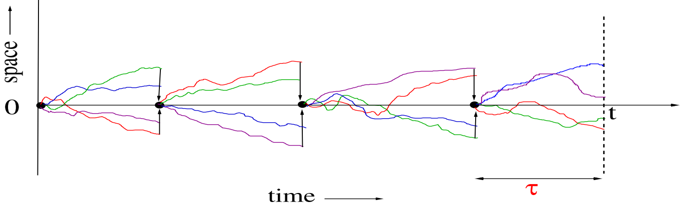

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Joint distribution at any time t:

$$P_r\left(\{x_i\}|t\right) = e^{-r\,t}\prod_{i=1}^{N} p_0(x_i|t) + r\int_0^t d\tau\, e^{-r\,\tau}\prod_{i=1}^{N} p_0(x_i|\tau)$$

where $p_0(x|\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$

M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

Dynamically emergent correlations

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The joint distribution does not **factorize** $\Longrightarrow$ **correlated** resetting gas

In this model, interactions between particles are **not built-in**, but the correlations are generated by the dynamics (**simultaneous resetting**), that persist all the way to the stationary state

$\longrightarrow$ **dynamically emergent correlations**

Dynamically emergent correlations

Joint distribution at any time t:

$$P_r(\{x_i\}|t) = e^{-r\,t} \prod_{i=1}^{N} p_0(x_i|t) + r \int_0^t d\tau\, e^{-r\,\tau} \prod_{i=1}^{N} p_0(x_i|\tau)$$

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Correlation function at time t for any pair $i \neq j$:

By symmetry, $\langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle = 0$ at all times t

Dynamically emergent correlations

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Correlation function at time t for any pair $i \neq j$:

By symmetry, $\langle x_i x_j\rangle - \langle x_i\rangle\langle x_j\rangle = 0$ at all times t

One needs to go to higher order to detect the correlations

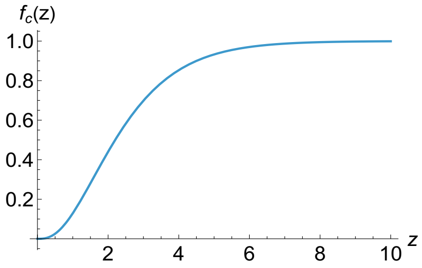
Dynamically emergent correlations

Higher order correlation function for $i \neq j$

$$C_{i,j}(t) = \langle x_i^2\, x_j^2 \rangle - \langle x_i^2 \rangle \langle x_j^2 \rangle = \frac{4D^2}{r^2}\, f_c(r\,t)$$

where

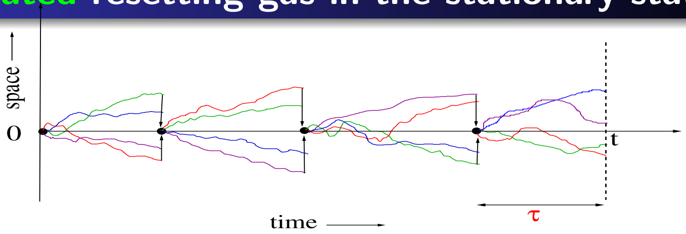
$$f_c(z) = 1 - 2\,z\,e^{-z} - e^{-2\,z}$$



$$f_c(z) \approx \begin{cases} \frac{1}{3}\, z^3 & z \to 0 \\ 1 - 2\,z\,e^{-z} & z \to \infty \end{cases}$$

correlations **grow with time**

Correlated resetting gas in the stationary state



N Brownian motions (independent) that are **simultaneously** reset with rate r to the origin

The joint position distribution approaches a nonequilibrium stationary state (NESS) at long times $t \to \infty$

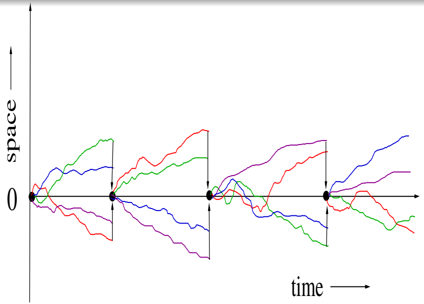
$$P_r^{\rm st}\left(\{x_i\}\right) = r \int_0^\infty d\tau\, e^{-r\,\tau} \prod_{i=1}^{N} \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

The joint distribution does not **factorize** even in the NESS

$\langle x_i^2 x_j^2 \rangle - \langle x_i^2 \rangle \langle x_j^2 \rangle = 4\frac{D^2}{r^2} \Longrightarrow$ **attractive all-to-all** interaction

$\Longrightarrow$ **strongly correlated** resetting gas

Solvable Correlated Gas



Joint distribution:

$$P_r^{\rm st}(\{x_i\}) = r\int_0^\infty d\tau\, e^{-r\tau}\prod_{i=1}^{N} p_0(x_i|\tau)$$

$$p_0(x|\tau) = \frac{1}{\sqrt{4\pi D\tau}}\, e^{-x_i^2/4D\tau}$$

The stationary joint distribution has a **CIID** structure $\Longrightarrow$ Solvable

$$\boxed{P_r^{\rm st}(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} du\, h(u)\prod_{i=1}^{N} p_0(x_i|u)}$$

Here $u \equiv \tau$ and $h(u) \equiv r\, e^{-r\,u}\,\theta(u)$

CIID $\Longrightarrow$ Conditionally Independent and Identically Distributed

Super-statistical ensembles in random matrix theory

Examples of deformed Gaussian random matrix ensembles:

$$P(\mathbf{X}) = \int_0^\infty du\, h(u)\, \exp\left[-\frac{1}{2\,u}\,\mathrm{Tr}(X^2)\right]$$

The variance $u \Longrightarrow$ random variable drawn from $h(u)$

Abdul-Magd (2005); Bohigas, Carvalho, Pato (2008);
Abdul-Magd, Akemann, Vivo (2009)

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However, no **microscopic dynamics** leading to these ensembles

Solvable Correlated Gas

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Solvable Correlated Gas

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(1) Compute any observable for the ideal gas $\Rightarrow$ I.I.D variables with distribution $p_0(x|\tau)$ parametrized by $\tau \Longrightarrow$ easy

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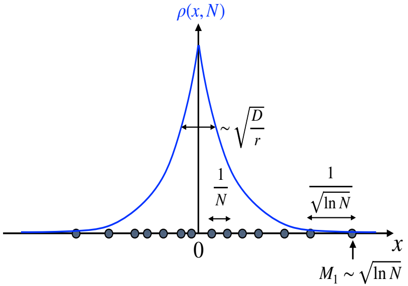
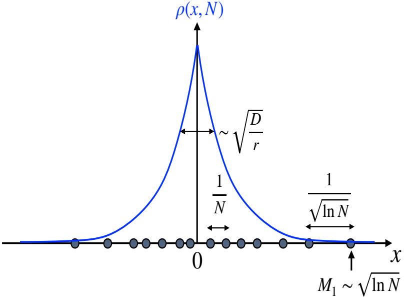


Examples:

- Average density
- Distribution of the k-th maximum: Order statistics
- Spacing distribution
- Full Counting Statistics

Explicit Results

Average Density



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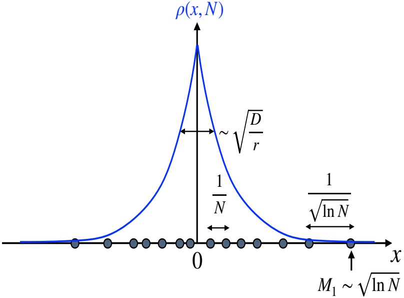
Average density:

$$\rho(x,N) = \frac{1}{N}\sum_{i=1}^{N}\langle\delta(x_i - x)\rangle = \int P_r^{\rm st}\left(x,\, x_2, \ldots, x_N\right)\, dx_2\, dx_3 \ldots dx_N$$

Average Density



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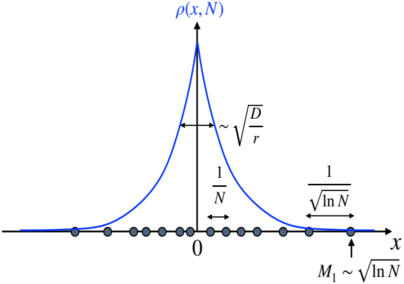
$$= r\int_0^\infty d\tau\, e^{-r\tau}\, p_0(x|\tau) = \frac{\alpha_0}{2}\,\exp[-\alpha_0\,|x|]$$

where $\alpha_0 = \sqrt{r/D}$

$\Longrightarrow$ same as the single particle position distribution

Order Statistics



$M_k \Longrightarrow k$-th maximum

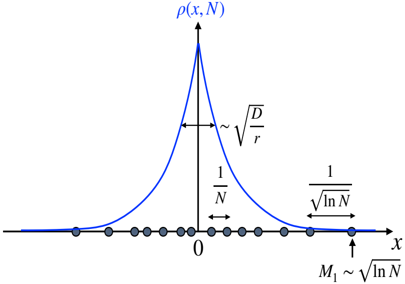
Set $k = \alpha N$

$\alpha \sim O(1) \Longrightarrow$ **bulk**

$\alpha \sim O(1/N) \Longrightarrow$ **edge**

Symmetric around $\alpha = \mathbf{1/2}$

Order Statistics



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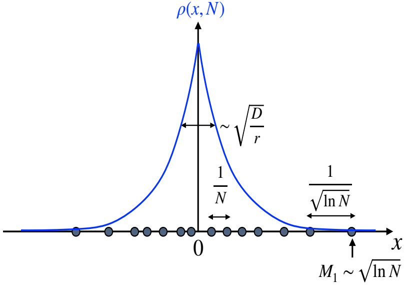
Order Statistics



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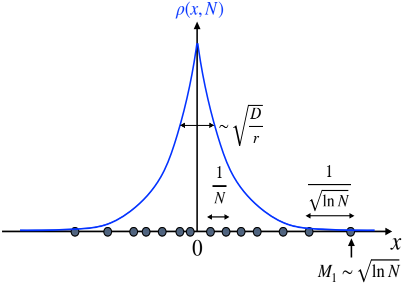

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M. Biroli, H. Larralde, S. M., G. Schehr, PRL, 130, 207101 (2023)

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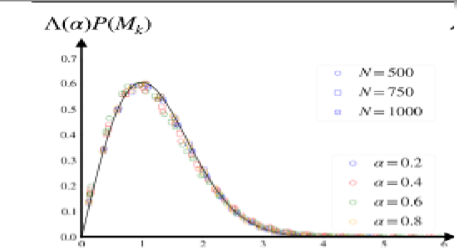

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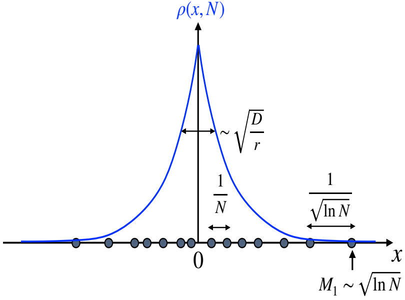
Gap/Spacing Statistics



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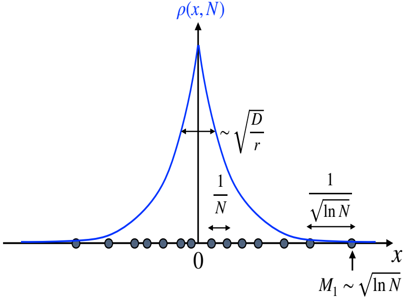

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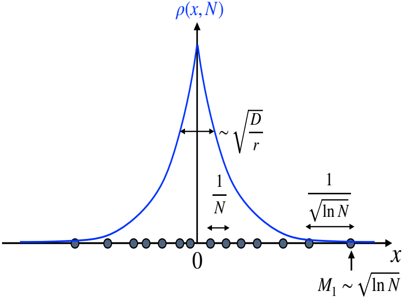

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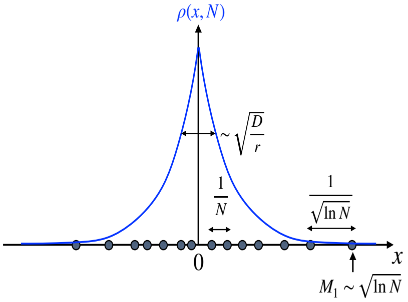

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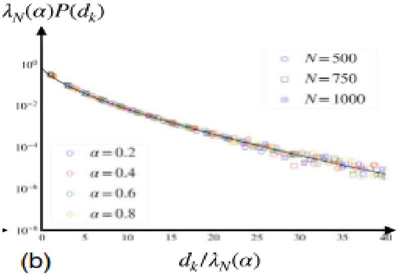
Gap/Spacing Statistics



The gap scaling function:

$$h_{\rm gap}(z) = 2\int_0^{\infty} du\, e^{-u^2 - z/u}$$

$\to \sqrt{\pi}$ as $z \to 0$

$\sim e^{-3(z/2)^{2/3}}$ as $z \to \infty$

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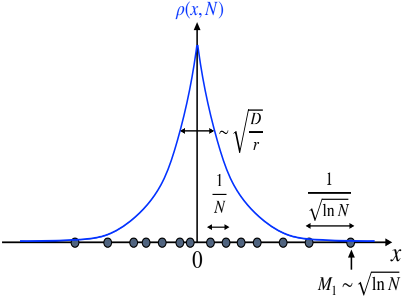
Full Counting Statistics



$N_L \Longrightarrow$ number of particles in $[-L, L]$

Clearly, $0 \leq N_L \leq N$

$P(N_L, N) = ?$

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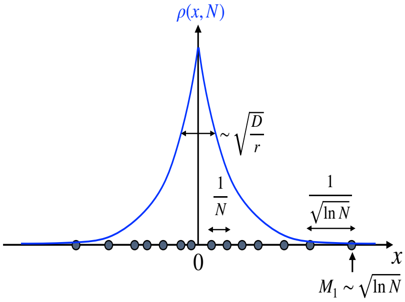

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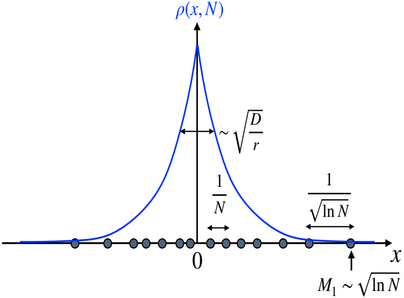

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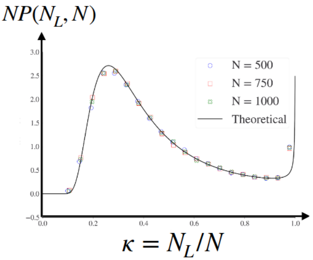
where the scaling function:

$$H(\kappa) = \gamma\sqrt{\pi}\,[u(\kappa)]^{-3}\,\exp\left[-\gamma\, u^{-2}(\kappa) + u^2(\kappa)\right]$$

with $\gamma = r\,L^2/(4D)$ and $u(\kappa) = \mathrm{erf}^{-1}(\kappa)$

Full Counting Statistics



The scaling function $H(\kappa)$

$$H(\kappa) \to \frac{8\gamma}{\pi\,\kappa^3}\ \exp\left[-\frac{4\gamma}{\pi\,\kappa^2}\right] \quad \text{as } \kappa \to 0$$

$$H(\kappa) \to \frac{\gamma\,\sqrt{\pi}}{(1-\kappa)\,[\ln(1-\kappa)]^{3/2}} \quad \text{as } \kappa \to 1$$

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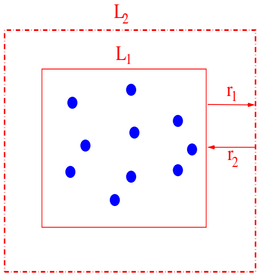
Summary so far . . .



A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

$r_1 \to \infty$ and $r_2 = \mathbf{r}$

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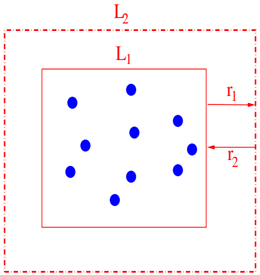


A limiting case:

$L_1 = 0$ and $L_2 \to \infty$

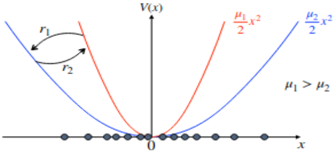
$r_1 \to \infty$ and $r_2 = \mathbf{r}$

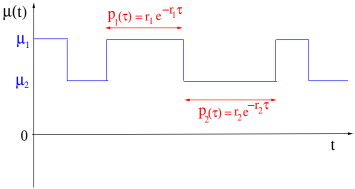
- This limiting case $\Longrightarrow$ **Exactly solvable** with a **strongly correlated NESS**
- The stationary joint distribution in the **NESS** $\Longrightarrow$ **CIID** structure

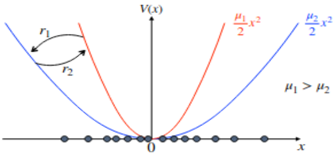
  $\Longrightarrow$ allows us to compute physical observables expliciitly
- How generic is the **CIID** structure $\Longrightarrow$ going beyond this limiting case ?

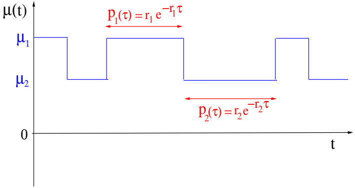
Switching Harmonic Trap

N Brownian particles in a switching harmonic trap







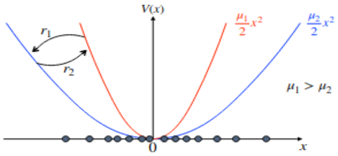


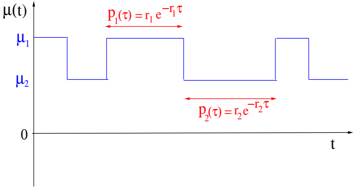
$$\frac{dx_i}{dt} = -\mu(t)\, x_i + \sqrt{2D}\, \eta_i(t)$$

$\eta_i(t) \longrightarrow$ Gaussian white noise with zero mean
and correlator $\langle \eta_i(t)\eta_j(t') \rangle = \delta_{i,j}\, \delta(t - t')$

The stiffness $\mu(t)$ of the harmonic trap changes from $\mu_1 \to \mu_2 < \mu_1$ with rate r_1 and $\mu_2 \to \mu_1$ with rate $r_2 \Longrightarrow$ dichotomous telegraphic noise

N Brownian particles in a switching harmonic trap





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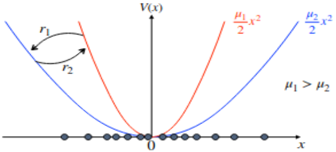
$\Longrightarrow$ drives the system into a correlated NESS with a stationary joint distribution $P(x_1, x_2, \ldots, x_N | t \to \infty) = P^{\rm st}(\vec{x}|t \to \infty) = ?$

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

N Brownian particles in a switching harmonic trap



The limit

$\mu_1 \to \infty$, $\mu_2 \to 0$,
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$\Longrightarrow$ simultaneous resetting model

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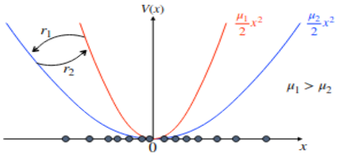

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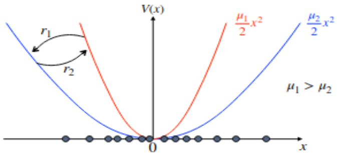

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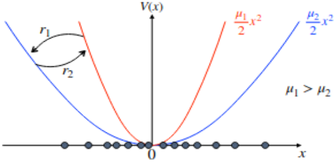
They satisfy a pair of coupled Fokker-Planck equations:

$$\partial_t P_1 = D \sum_{i=1}^{N} \partial_{x_i}^2 P_1 + \mu_1 \sum_{i=1}^{N} \partial_{x_i} (x_i\, P_1) - r_1\, P_1 + r_2\, P_2$$

$$\partial_t P_2 = D \sum_{i=1}^{N} \partial_{x_i}^2 P_2 + \mu_2 \sum_{i=1}^{N} \partial_{x_i} (x_i\, P_2) - r_2\, P_2 + r_1\, P_1$$

with initial conditions: $P_1(\vec{x}|0) = \frac{1}{2}\,\delta(\vec{x})$ and $P_2(\vec{x}|0) = \frac{1}{2}\,\delta(\vec{x})$

Exact stationary solution



Fourier transforms:

$$\tilde{P}_{1,2}(\vec{k}|t) = \int P_{1,2}(\vec{x}|t)\, e^{i\,\vec{k}\cdot\vec{x}}\, d\vec{x}$$

Rotational symmetry

$$\Longrightarrow \tilde{P}_{1,2}(\vec{k}|t) = \tilde{P}_{1,2}(k|t)$$

where
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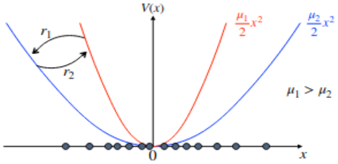

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where
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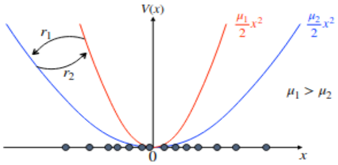
Exact **stationary** solution in terms of $R_1 = \frac{r_1}{2\mu_1}$ and $R_2 = \frac{r_2}{2\mu_2}$

$$\tilde{P}_1^{\rm st}(k) = \frac{r_2}{r_1+r_2}\, e^{-D\,k^2/(2\mu_1)}\, M\left(R_1, 1+R_1+R_2, -\frac{D\,k^2\,(\mu_1-\mu_2)}{2\,\mu_1\,\mu_2}\right)$$
$$\tilde{P}_2^{\rm st}(k) = \frac{r_1}{r_1+r_2}\, e^{-D\,k^2/(2\mu_2)}\, M\left(R_2, 1+R_1+R_2, -\frac{D\,k^2\,(\mu_2-\mu_1)}{2\,\mu_1\,\mu_2}\right)$$

where $M(a,b,z) = 1 + \frac{a}{b}\,z + \frac{a(a+1)}{b(b+1)}\,\frac{z^2}{2!} + \cdots \longrightarrow$ Kummer's function

$R_1 = \frac{r_1}{2\mu_1}$ and $R_2 = \frac{r_2}{2\mu_2}$

Exact stationary solution



Fourier transforms:

$\tilde{P}_{1,2}(\vec{k}|t) = \int P_{1,2}(\vec{x}|t)\, e^{i\,\vec{k}\cdot\vec{x}}\, d\vec{x}$

Rotational symmetry

$\Longrightarrow \tilde{P}_{1,2}(\vec{k}|t) = \tilde{P}_{1,2}(k|t)$

where

$k^2 = k_1^2 + k_2^2 + \ldots + k_N^2$

Exact **stationary** solution in terms of $R_1 = \frac{r_1}{2\mu_1}$ and $R_2 = \frac{r_2}{2\mu_2}$

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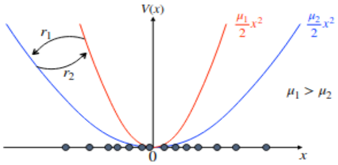

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The full solution: $\tilde{P}^{\rm st}(k) = \tilde{P}_1^{\rm st}(k) + \tilde{P}_2^{\rm st}(k)$

Inverse Fourier transform $P^{\rm st}(\vec{x})$ has a CIID structure $\longrightarrow$ not **manifest**

Fourier inversion

$$\tilde{P}_1^{\rm st}(k) = \frac{r_2}{r_1+r_2}\, e^{-D\,k^2/(2\mu_1)}\, M\left(R_1, 1+R_1+R_2, -\frac{D\,k^2\,(\mu_1-\mu_2)}{2\,\mu_1\,\mu_2}\right)$$

$$\tilde{P}_2^{\rm st}(k) = \frac{r_1}{r_1+r_2}\, e^{-D\,k^2/(2\mu_2)}\, M\left(R_2, 1+R_1+R_2, -\frac{D\,k^2\,(\mu_2-\mu_1)}{2\,\mu_1\,\mu_2}\right)$$

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Using the integral representation

$$M(a, b, -c\,k^2) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 du\, u^{a-1}\,(1-u)^{b-a-1}\, e^{-c\,k^2\,u}$$

One can invert $\tilde{P}^{\rm st}(k) = \tilde{P}_1^{\rm st}(k) + \tilde{P}_2^{\rm st}(k)$ explicitly $\Longrightarrow$

'Hidden' CIID structure of the stationary state

Inverting the Fourier transform one finds the **CIID** representation

$$P^{\mathrm{st}}(\vec{x}) = \int_0^1 du\, h(u) \prod_{i=1}^{N} \frac{1}{\sqrt{2\, V(u)}}\, e^{-x_i^2/(2\, V(u))}$$

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where

$$V(u) = D \left(\frac{u}{\mu_2} + \frac{1-u}{\mu_1} \right)$$

and

$$h(u) = A\, u^{R_1 - 1}\, (1-u)^{R_2 - 1} \left[\frac{u}{\mu_2} + \frac{1-u}{\mu_1} \right]$$

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$$h(u) = A\, u^{R_1-1}\,(1-u)^{R_2-1}\,\left[\frac{u}{\mu_2} + \frac{1-u}{\mu_1}\right]$$

Since $\int_0^1 h(u)du = 1$, the function $h(u)$ can be interpreted as the PDF of the random variable $u \in [0, 1]$ $\longrightarrow$ the fraction of time each particle spends in μ_2 phase

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

All observables $\longrightarrow$ exactly solvable

Using the explicit **CIID** structure of the stationary joint PDF

$$P^{\rm st}(\vec{x}) = \int_0^1 du\, h(u) \prod_{i=1}^{N} p_0(x_i|u)$$

all observables in the correlated **NESS** can be computed explicitly and they exhibit rich and interesting behaviors

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

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For example, the extreme value statistics (**EVS**):

$$P(M_1 = w, N) \to \frac{1}{\sqrt{\ln N}}\, f\left(\frac{w}{\sqrt{\ln N}}\right)$$

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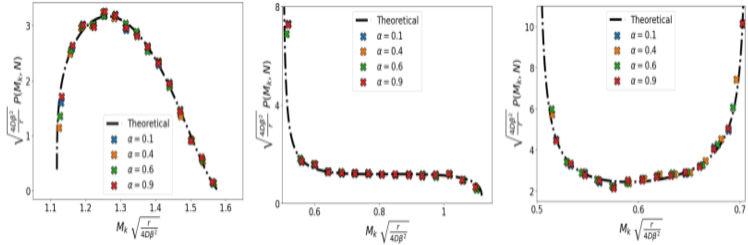
For example, the extreme value statistics (**EVS**):

$$P(M_1 = w, N) \to \frac{1}{\sqrt{\ln N}}\, f\left(\frac{w}{\sqrt{\ln N}}\right)$$

where the exact scaling function (with $R_1 = \frac{r_1}{2\mu_1}$ and $R_2 = \frac{r_2}{2\mu_2}$):

$$f(z) = B\, z^3 \left(1 - \frac{z^2}{R_2}\right)^{R_2 - 1} \left(\frac{z^2}{R_1} - 1\right)^{R_1 - 1} \text{ with } \sqrt{R_1} \le z \le \sqrt{R_2}$$

$\longrightarrow$ a new extreme value distribution of **strongly** correlated random variables with a **finite** support



The exact scaling function for the distribution of the scaled k-th maximum M_k

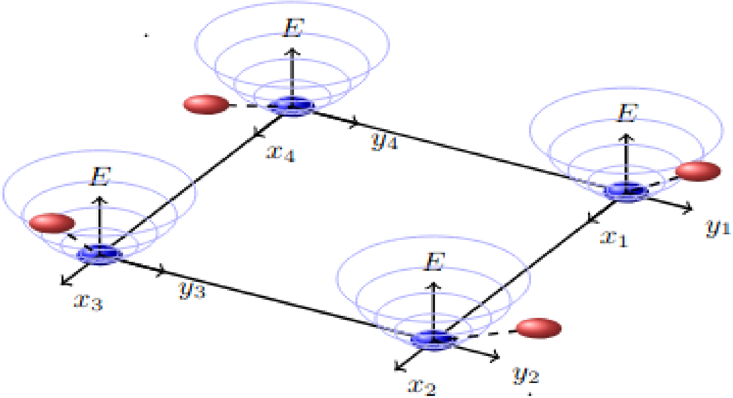
$$f(z) = B\, z^3 \left(1 - \frac{z^2}{R_2}\right)^{R_2 - 1} \left(\frac{z^2}{R_1} - 1\right)^{R_1 - 1} \text{ with } \sqrt{R_1} \le z \le \sqrt{R_2}$$

The scaling function $f(z) \longrightarrow$ **universal**, i.e,., same for all M_k's in $d = 1$ and also for all $\mathbf{d \geq 1}$

Biroli, Kulkarni, S.M., Schehr, PRE, 109, L032106 (2024)

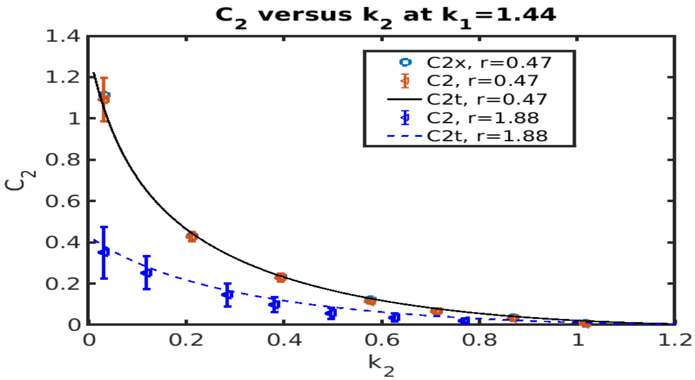
Experimental results:

[set-up: S. Ciliberto]



- colloidal particles in synchronized harmonic traps
- particles are immersed in fluid $\Longrightarrow$ long-range hydrodynamic interaction (**neglected** in the theoretical model)

Experimental results on C_2

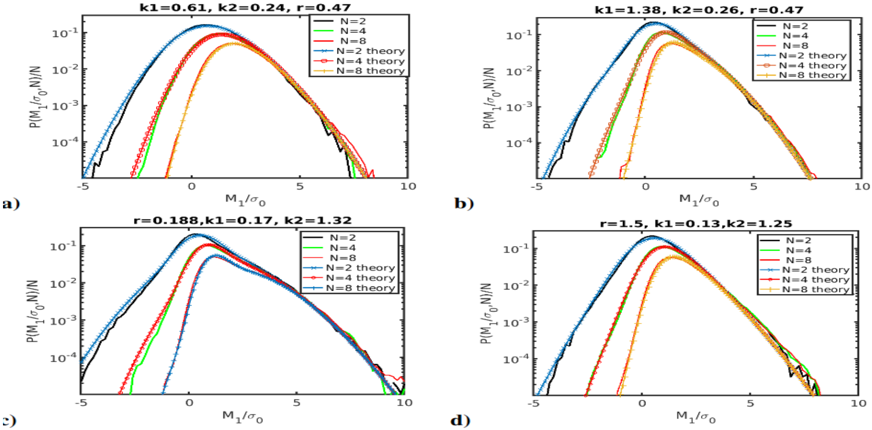


$$C_2 = \frac{\langle x_i^2 \, x_j^2 \rangle}{\langle x_i^2 \rangle \langle x_j^2 \rangle} - 1$$ [S. Ciliberto, unpublished data]

Theoretical prediction [for $r_1 = r_2 = r$ and with $R_1 = \frac{r}{2\mu_1}$ and $R_2 = \frac{r}{2\mu_2}$]:

$$C_2 = \frac{(\mu_2 - \mu_1)^2 (2 + 3R_1 + 3R_2 + 4R_1 R_2)}{(2 + R_1 + R_2)(2r + \mu_1 + \mu_2)^2}$$

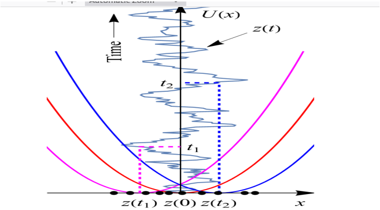
Experimental results on M_1:

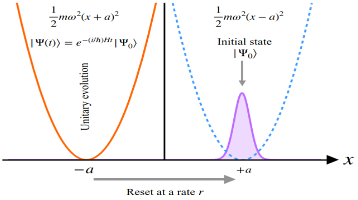


Experiments with a finite number of colloidal particles in an optical trap

$\Longrightarrow$ up to $N = 8$ particles [S. Ciliberto, unpublished data]

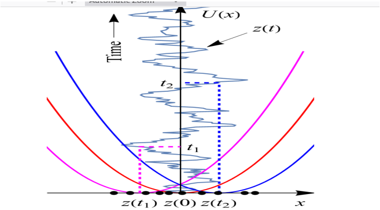
Two other models with CIID structure

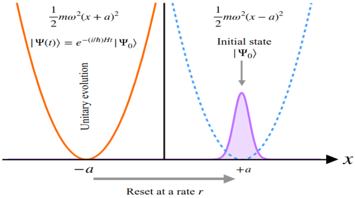




N noninteracting particles (bosons) in a harmonic trap

Two other models with CIID structure



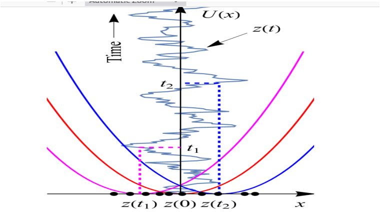


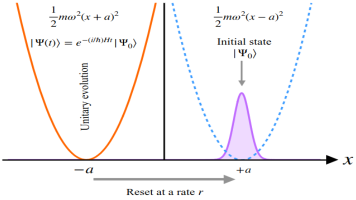
N noninteracting particles (bosons) in a harmonic trap

(1) Model 1 (Classical): The center of the harmonic trap performs a stochastic motion $\Longrightarrow$ drives the system into a correlated NESS

Sabhapandit & S.M. J. Phys. A.: Math. Theor. **57**, 335003 (2024)

Two other models with CIID structure





N noninteracting particles (bosons) in a harmonic trap

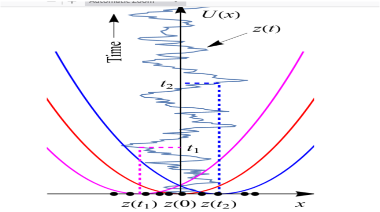
(1) Model 1 (Classical): The center of the harmonic trap performs a stochastic motion $\Longrightarrow$ drives the system into a correlated NESS

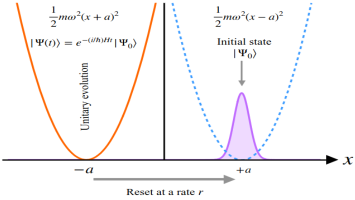
Sabhapandit & S.M. J. Phys. A.: Math. Theor. **57**, 335003 (2024)

(2) Model 2 (Quantum): N noninteracting bosons in the ground state of a harmonic trap whose center is quenched from $+a$ to $-a$, evolves unitarily for a random time and then the state is reset to the ground state with center at $+a$ $\Longrightarrow$ drives the system into a correlated NESS

Kulkarni, S.M. & Sabhapandit, J. Phys. A: Math. Theor. **58**, 105003 (2025)

Two other models with CIID structure





In both models, the NESS has the **CIID** (conditionally independent and identically distributed) structure

$$P^{\rm st}(x_1, x_2, \ldots, x_N) = \int_{-\infty}^{\infty} du\, h(u) \prod_{i=1}^{N} p_0(x_i|u)$$

This **CIID** structure makes the problem **solvable** for various observables such as average density, spacing distribution, extreme statistics, full counting statistics etc.

Summary and Conclusion

- **Stochastic fluctuation** of the **common** environment

  $\Longrightarrow$ **strong emerging** correlations between particles

- Exactly solvable example: **switching** harmonic trap

- The NESS has a **CIID** structure

  $\Longrightarrow$ Several physical observables are exactly computable and have rich interesting behaviors, despite being a **strongly correlated** system

- Comparison with experiments on colloidal particles in an optical trap

- Easily generalisable to a whole new class of solvable correlated gases in their nonequilibrium stationary state $\longrightarrow$ ballistic particles, Lévy flights, harmonic potential with a stochastic center, noninteracting bosons, ..

  $\Longrightarrow$ all have this **CIID** structure $\Rightarrow$ **Exactly solvable**

Collaborators

- Marco Biroli (LPTMS, Univ. Paris Saclay)
- Sergio Ciliberto (ENS, Lyon)
- Manas Kulkarni (ICTS, Bangalore)
- Hernan Larralde (UNAM, Mexico)
- Gabriele de Mauro (LPTMS, Orsay)
- Nikhil Mesquita (RRI, Bangalore)
- Sanjib Sabhapandit (RRI, Bangalore)
- Gregory Schehr (LPTHE, Univ. Sorbonne)

Earlier work on **Stochastic Resetting** with

- M. R. Evans (Edinburgh, UK)

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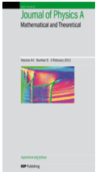

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